

SECOND EDITION

# Earth System Modeling, Data Assimilation and Predictability

Atmosphere, Oceans, Land and Human Systems



Eugenia Kalnay • Safa Mote • Cheng Da

## Earth System Modeling, Data Assimilation and Predictability

Second Edition

Since the publication of the first edition of this highly regarded textbook, the value of data assimilation has become widely recognized across the Earth sciences and beyond. Data assimilation methods are now being applied to many areas of prediction and forecasting, including extreme weather events, wildfires, infectious disease epidemics, and economic modeling. This second edition provides a broad introduction to applications across the Earth systems and coupled Earth–human systems, with an expanded range of topics covering the latest developments of variational, ensemble, and hybrid data assimilation methods. New toy models and intermediate-complexity atmospheric general circulation models provide hands-on engagement with key concepts in numerical weather prediction, data assimilation, and predictability. The inclusion of computational projects, exercises, lecture notes, teaching slides, and sample exams makes this textbook an indispensable and practical resource for advanced undergraduate and graduate students, researchers, and practitioners who work in weather forecasting and climate prediction.

**Eugenia Kalnay** completed her PhD at the Massachusetts Institute of Technology (MIT) under Jule Charney and became the first woman on the faculty in the Department of Meteorology. In 1979, she moved to NASA's Goddard Space Flight Center, where she developed the fourth-order global numerical model and led experiments in the new science called "data assimilation." In 1984, she became Head of NASA's Global Modeling and Simulation Branch. In 1987, she became Director of the National Oceanic and Atmospheric Administration's Environmental Modeling Center, where many improvements of models and data assimilation were developed for the National Weather Service forecasts. Her paper "The NCEP/NCAR 40-year reanalysis project" (Kalnay et al., 1996) is the most cited paper in geosciences. In 1997, Kalnay became Lowry Chair at the University of Oklahoma and in 1999 became Atmospheric and Ocean Sciences Department Chair and professor at the University of Maryland, where she was later elected a Distinguished University Professor.

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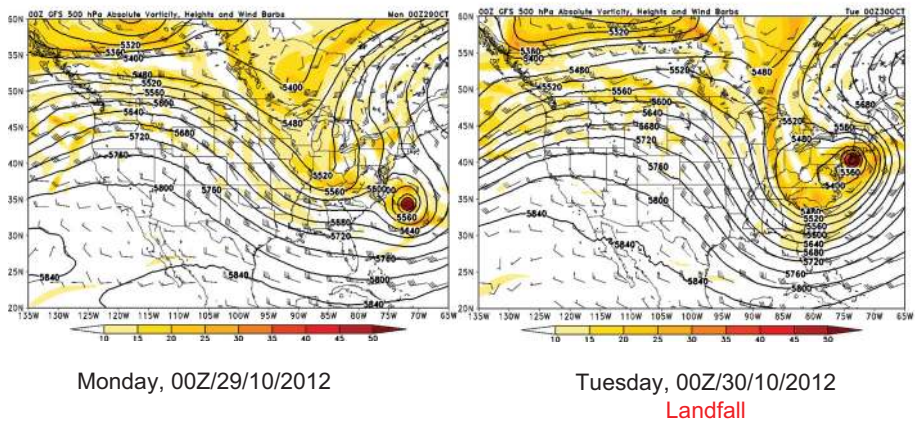
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**Front Cover Legend:** Importance of ensemble forecasting and data assimilation for the forecast of the Hurricane Sandy landing in the New York metro area in 2012 (The actual trajectory of Sandy is shown with a thick black line with a black circle every 6 hours indicating when a new “assimilation of the observation of Sandy’s location” took place). This figure, created by Clark Evans, Professor of Atmospheric Sciences at the University of Wisconsin–Milwaukee, (evans36@uwm.edu), shows the National Weather Service ensemble prediction of the devastating Sandy Hurricane landing.

Note that the ensemble trajectories of the NCEP 10-day ensemble forecasts of the Sandy hurricane trajectory were started on October 23, 12UTC (shown with blue lines at the first identification of Sandy), and 6 and 12 hours later (shown with green and red lines respectively). The majority of the earliest (in blue) ensemble forecasts miss the hurricane being “captured” by a strong atmospheric trough (see the inside cover of the book) and continue moving eastwards, driven by the Atlantic westerly winds, as the majority of the Atlantic hurricanes normally do. The green trajectories, that after 6 hours underwent one additional data assimilation, clearly turned west, indicating the influence of the trough that captured the hurricane. The red trajectories, started October 24 00UTC, from the next data assimilation just 12 hrs after the blue trajectories, have a majority that correctly turns west for the landfall.

Why did Sandy turn west around 00Z/29/10/2012?  
It was captured by a deep trough!



500 hPa NCEP analysis of absolute vorticity, winds, and heights

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